Activity 19 Solving trigonometric equations

Aim: Construct an interactive unit circle and use it to solve trigonometric equations.

Setup the eActivity to solve $\sin\theta = k$ and $\cos\theta = k$

Setup the eActivity

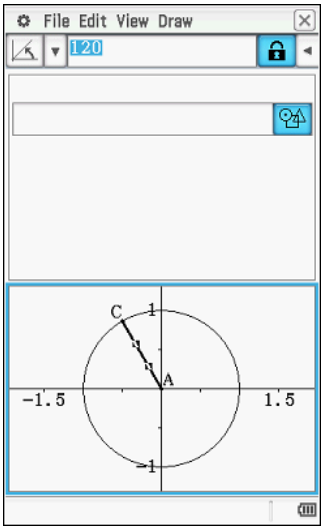
- Open the eActivity application
- Insert a Geometry Strip
- Select [⚙ | Geometry Format]
  - Set Measure Angle to Degree
  - Set Grid to Off

Construct the unit circle

- Construct a circle
- Constrain the centre of the circle at (0,0)
- Constrain the radius to one unit
- Hide the point on the circumference

Draw a radial line segment

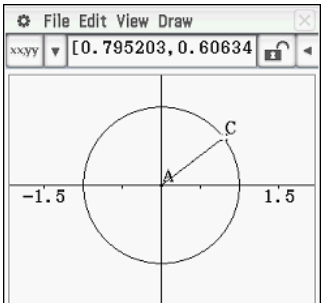
- Place a new point on the circumference
- Draw a line segment from the centre of the circle to the point on the circumference
- Tap ▶ to go round the corner
- Select the line segment
- Select 📐 from the Measure pull-down menu
- Type over the angle (eg. 120 EXE) and watch the line segment change direction



1.

a) Consider the equation $\sin\theta = 0.6$.

- Tap ▶ to go round the corner
- Tap point C and drag it to a point on the circle where the vertical co-ordinate is as close as possible to 0.6
- Tap the line segment
- Select 📐 from the Measure pull-down menu



What is one solution to the equation $\sin\theta = 0.6$?

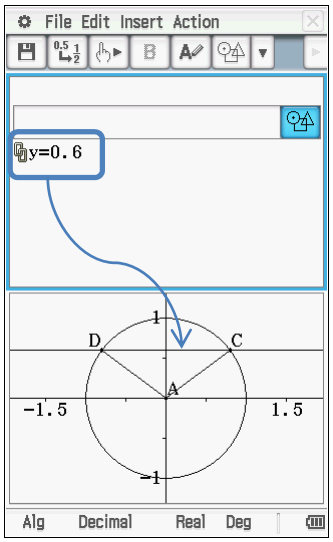
b) Is there another point on the circle with a vertical co-ordinate of 0.6? Move point C to this point then note the angle.

c) What is the relationship between the angles in a) and b)? Can this be generalised for all equations of the form $\sin\theta = k$?

d) Does the relationship in c) hold for negative values of k? Investigate.

Solve with Geometry link a more efficient method

- In the Geometry window, tap to select the line segment and press [Clear] to delete it
- Similarly, delete point C
- Tap in the eActivity window and insert a Geometry Link
- Type the equation $y = 0.6$
- Highlight the equation then drag and drop it into the Geometry window
- Tap to select the line and the circle, then select ⊠ from the Construct pull down menu to place points at the intersections
- Draw line segments AC and AD
- Tap a line segment then tap ▶ to go round the corner. The Measure window should display the angle.
- Repeat for the other line segment to find the other angle



2. Change the equation in the Geometry Link to $y = 0.3$ and press [EXE]. Use the resulting unit circle diagram to solve the equation $\sin\theta = 0.3$ for $0° \le \theta \le 360°$.

3. Solve the equation $\sin\theta = -0.5$ for $0° \le \theta \le 360°$.

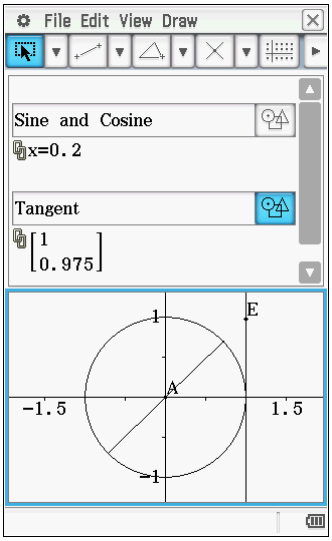
Recall that the cosine of an angle is the horizontal ordinate of the point of intersection of the line segment and the circle.

4. Change the equation in the Geometry Link to help you solve the equation $\cos\theta = 0.2$ for $0° \le \theta \le 360°$

5. Explain how you could extend your answer to Q4 if the domain was changed to $-360° \le \theta \le 720°$.

Setup the eActivity to solve $\tan\theta = k$

- Tap in the eActivity window to make it active
- Insert a new Geometry Strip
- Open your file for the Tangent ratio from the Activity *Unit circle* from the Mathematics Methods Units 1&2 text. See Learning notes for instructions if you have not completed that activity
- Tap in the eActivity window and insert a new Geometry Link
- Finally, click the point on the tangent and drag it into the Geometry Link



We can now change the *y*-ordinate of the point on the tangent line and see the corresponding change in the angles in the unit circle.

6.

a) Change the *y* value to 0.7 and determine the angles of the two line segments.

b) What is the relationship between the two angles in a)?

7. Solve the equation $\tan\theta = -1$ for $0° \le \theta \le 360°$.

8. Explain how you could extend your answer to Q7 if the domain was changed to $0° \le \theta \le 720°$.

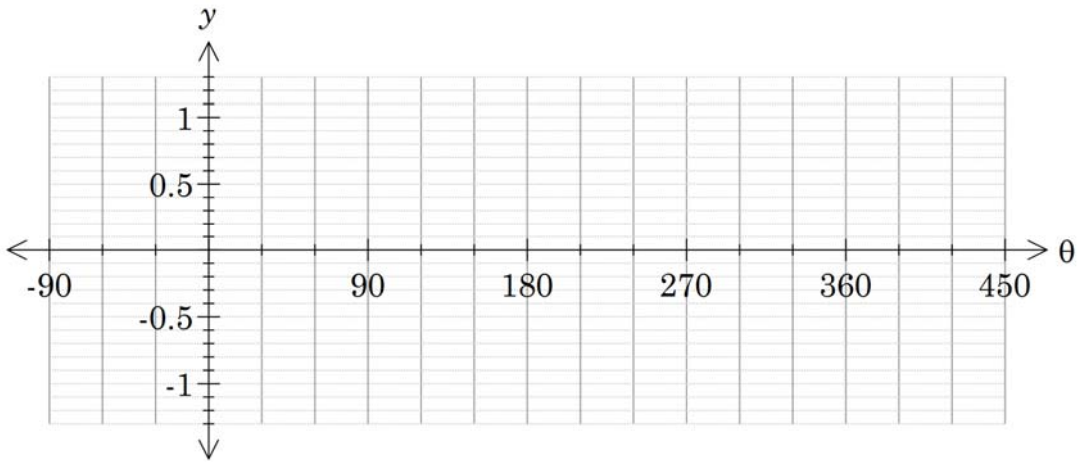
Multiple angles

9. Consider the equation $\sin 2\theta = 0.8$ for $0° \le \theta \le 360°$. Use your eActivity to help you find all the values of θ within the domain that make this equation true. Are you sure you have found them all?

10. Check your answer to Q9 using solve in Main.

11.

a) On the axes below, graph the functions $y = \sin(2\theta)$ and $y = 0.8$.



b) Show the solutions to the equation $\sin 2\theta = 0.8$ for $0° \le \theta \le 360°$ from Q9 on your graph above.

12. Use a graphical approach to solve the equation

$-2\cos\left(x+\frac{\pi}{3}\right)=1$ for $-\pi \le x \le \pi$.

Learning notes

Construction of the Tangent eActivity

Setup the eActivity

- Tap in the eActivity window to make it active
- Insert a new Geometry Strip

Construct the unit circle

- Construct a circle
- Constrain the centre of the circle at (0,0)
- Constrain the radius to one unit
- Hide the point on the circumference

Draw a radial line

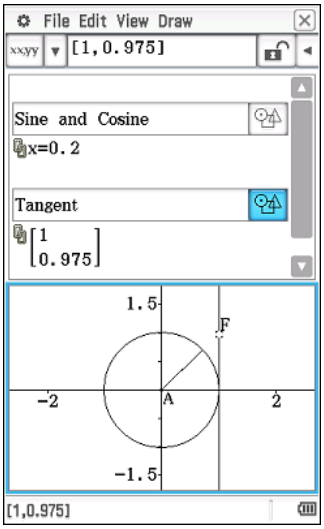
- Place a new point on the circumference
- Draw a line segment from the centre of the circle to the point on the circumference
- Hide the point on the circumference

Insert the tangent line

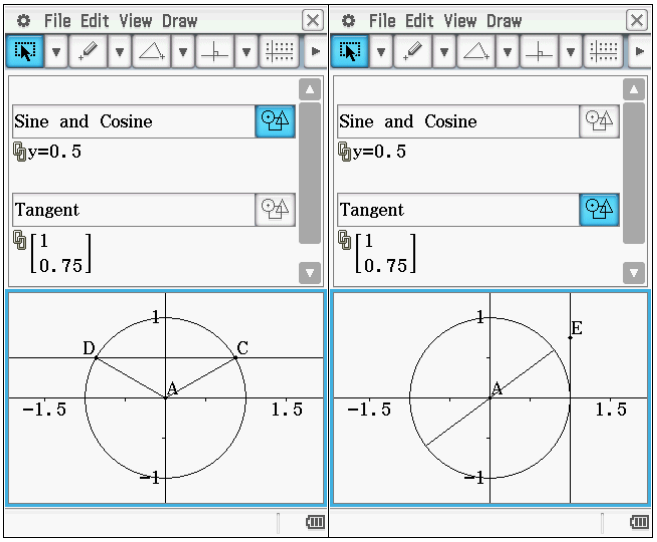
- Select the infinite line tool from the Draw pull down menu
- Tap twice in two different places on the canvas to insert a line
- Select the line then tap to go round the corner
- Set the equation of the line to $x = 1$
- Hide the points on the line

Locate the intersection

- Select the line and the line segment
- Tap or select [Draw | Construct | Intersect]
- Tap in the eActivity window
- Select [Insert | Geometry Link]
- Click the point of intersection and drag it into the Geometry Link



Once you have completed this activity, you can tidy up the eActivity with annotations such as those shown below. This can then be saved using [File | Save] and choosing an appropriate name.



Note the use of multiple strips. Individual Geometry diagrams can be opened by tapping the button at the end of the strip.

By tapping in the Geometry Strip and selecting [Insert | Add Strip Help] you can type some text that explains the Geometry diagram in more detail.

Multiple angles

When solving trigonometric equations involving multiple angles, you are encouraged to alter the domain accordingly, for example, the equation $\cos 3\theta = 0.7$ for $0° \le \theta \le 180°$ would become $\cos 3\theta = 0.7$ for $0° \le 3\theta \le 540°$. We then find solutions for 3θ in the given domain, then divide all answers by 3 to solve for θ.